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# Radion couplings to bulk fields in the Randall-Sundrum model*

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Abstract: The radion may be the lightest new state present in the Randall-Sundrum(RS) model. We examine the couplings of the radion to the Standard Model(SM) fields in the scenario where they propagate in the bulk and expand into Kaluza-Klein towers. These couplings are then contrasted with those of the more familiar case where the SM fields are confined to the TeV-brane. We find that the couplings of the radion to both $gg$ and $\gamma\gamma$ can be significantly different in these two cases. Implications for radion collider phenomenology are discussed.

Keywords: Extra Large Dimensions, Beyond Standard Model, Higgs Physics.

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## Contents



## 1. Introduction

Theories with extra dimensions provide a new way to attack the hierarchy problem. An exciting feature of these scenarios is that they lead to concrete and quite distinctive phenomenological tests. If such theories truly describe the source of the observed hierarchy, then their direct signatures should appear in future collider experiments that probe the TeV scale. In the specific model of Randall and Sundrum (RS) [1], the observed hierarchy is created by an exponential warp factor which arises from a 5-dimensional non-factorizable geometry. The collider signatures for this model have been studied in detail in [2].

The RS setup consists of a 5-dimensional non-factorizable geometry based on a slice of $AdS_5$ space with length $\pi r_c$, where $r_c$ denotes the compactification radius. Two 3-branes, with equal and opposite tensions, rigidly reside at the $\mathbb{S}_1/\mathbb{Z}_2$ orbifold fixed points at the boundaries of the $AdS_5$ slice, taken to be $y = r_c\phi = 0, r_c\pi$. The 5-dimensional Einstein's equations permit a solution which preserves 4-dimensional Poincaré invariance with the metric

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2 \,, \tag{1.1}$$

where the Greek indices extend over ordinary 4-d spacetime and the warp factor is given by $\sigma(\phi) = kr_c|\phi|$. Here $k$ is the $AdS_5$ curvature scale which is of order the Planck mass and is determined by the bulk cosmological constant $\Lambda = -24M_5^3k^2$, where $M_5$ is the 5-dimensional Planck mass that appears in the RS action. The 5-d curvature scalar is then given by $R_5 = -20k^2$; the requirement that quantum corrections be small, $|R_5| < M_5^2$, implies $k/M_{pl} \lesssim 0.1$. Examination of the action in the 4-d effective theory yields the relation $M_{pl}^2 = M_5^3/k(1 - e^{-2kr_c\pi}) \simeq M_5^3/k$ for the reduced 4-d Planck mass. The scale of physical phenomena as realized by the 4-d flat metric transverse to the 5th dimension $y = r_c\phi$ is specified by the exponential warp factor. TeV scales can naturally be attained on the



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3-brane at $\phi = \pi$ if gravity is localized on the Planck brane at $\phi = 0$ and $kr_c \simeq 11$–$12$. (We take $kr_c = 11.27$ in our numerical analysis below.) The scale of physical processes on this SM or TeV-brane is then $\Lambda_\pi \equiv M_{pl} e^{-kr_c\pi}$. The observed hierarchy is thus generated by a geometrical exponential factor and no other additional large hierarchies appear in the model.

That the quantity $kr_c$ can be stabilized and the above range of values can be made natural has been demonstrated by a number of authors [3] and leads directly to the existence of a massive radion($r$), which corresponds to a quantum excitation of the brane separation. In the original RS scenario, SM matter was confined to the TeV-brane. In this case it can be shown that the radion couples to the trace of the stress-energy tensor of the SM wall fields with a strength $\Lambda_r = \sqrt{3}\Lambda_\pi$ which is of order the TeV scale, i.e. $\mathcal{L}_{\rm eff} = -r\ T^\mu_\mu/\Lambda_r$. This leads to gauge and matter couplings for the radion that are qualitatively similar to those of the SM Higgs boson. The radion mass ($m_r$), which arises dynamically during the stabilization procedure, is expected to be significantly below the scale $\Lambda_r$ implying that the radion may be the lightest new field present in the RS model. One may expect on general grounds that this mass should lie in the range of a few $\times 10\,\mathrm{GeV} \le m_r \le \Lambda_r$. The phenomenology of the RS radion coupled to SM wall fields on the TeV-brane has been examined by a number of authors [4] and in particular has been recently reviewed by Kribs [5] to which the interested reader should refer for details.

For model building reasons one may consider placing some if not all of the SM gauge and fermion fields in the RS bulk; this possibility has been systematically examined by a number of authors [2, 6] in the non-recoil limit. While it has been shown that it is possible to consistently place SM gauge bosons and fermions in the bulk, for phenomenological reasons the Higgs fields which induce $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ breaking must remain on the TeV-brane. In such a scenario one may wonder how the radion couplings with the zero-modes of the bulk fields, which we now identify with the observed SM particles, may be modified from those which occur when the SM fields are on the TeV-brane. The purpose of this paper is to examine this issue in detail and elucidate the phenomenology of radions coupling to the bulk SM fields. Since the radion is expected to be the lightest new particle in the RS model, its couplings to the zero-modes of these fields will be the first probe of the detailed structure of the model. In order to isolate which new effects are due to "bulk versus brane" couplings, we will assume that all mixing among the Kaluza-Klein(KK) levels of bulk states is sufficiently small as to be negligible.

## 2. Bulk couplings

When the SM fields are constrained to lie on the TeV-brane their couplings to the radion only arise through spontaneous symmetry breaking (SSB), i.e. through the induced masses of the fermions and gauge bosons. In the absence of such terms the trace of the 4-d stress-energy tensor would receive zero contribution from these sectors. These Higgs-induced terms can be generically written as

$$S_{\rm wall} = \int d^4x\ \frac{-r}{\Lambda_r}\left[m_f \bar{f} f - M_V^2 V_\mu V^\mu\right], \tag{2.1}$$

where $f$ is any SM fermion and $V = W, Z$. In contrast to this situation, when the SM gauge and fermion fields are placed in the bulk, radion couplings can arise from a number of sources which we will investigate below. The terms of the stress-energy tensor to which the radion eventually couples now receive contributions from both bulk and brane terms in the full action. However, we note that with the Higgs field remaining on the TeV-brane the gauge and fermion mass terms arising from spontaneous symmetry breaking remain wall terms in the action as they are above. In the case where the SM fields are in the bulk, SSB must still induce masses for the zero modes of these fields which coincide with their SM values. This means that $S_{\rm wall}$ will be present in either case and its existence is not sensitive to the placement of the fermion and and gauge fields. Similarly, from these arguments it is obvious that the radion coupling to pairs of Higgs bosons will be left unaltered by the considerations below since the Higgs remains on the TeV-brane. With this in mind we now turn to the *pure* bulk terms and ignore those arising from the wall except when they are added to complete the final coupling expressions. In effect, this implies that we can examine the bulk terms in the absence of any SSB contributions.

In order to formulate the couplings of the radion to the bulk SM fields we must obtain expressions for the dynamical components of the metric tensor for the full RS model in 5-dimensions in terms of physical fields. This has recently been studied in detail in [7] to which we refer the reader for details. Removing the pure graviton tower excitation pieces from the dynamical parts of the metric yields the radion contributions. To linear order in the fields, the $\mu\nu$-components of the relevant terms in the metric are found to be

$$g^r_{\mu\nu} = -e^{2kr_ct}\left[\frac{\left(\pi\epsilon^2+t\right)\epsilon^{-1}}{\sqrt{6}\pi M_{pl}}\eta_{\mu\nu}\, r + \frac{\tilde{S}(t)\epsilon^{-1}}{\sqrt{24}\pi k r_c M_{pl}}\partial_\mu\partial_\nu r\right], \tag{2.2}$$

where $\epsilon = e^{-kr_c\pi}$, $t = |\phi|$ and $r$ is the $\phi$-independent radion field. The function $\tilde{S}(t)$ is defined by [7]

$$\tilde{S}(t) = \sum_{n=1}\kappa_n\left[\chi^{(n)}(t) - \chi^{(n)}(\pi)\right], \tag{2.3}$$

where the $\chi^{(n)}(t)$ are the familiar wave functions for the RS graviton tower KK excitations [2]:

$$\chi^{(n)}(t) \simeq \left(\sqrt{kr_c}\epsilon^{-1}\right) z^2 J_2\frac{(x_n z)}{J_2(x_n)}, \tag{2.4}$$

where $J_2$ is a Bessel function, $z = e^{kr_c(t-\pi)}$, with $x_n$ being the roots of $J_1$ and the corresponding $Y_2$ terms have been dropped for simplicity of notation. The coefficients $\kappa_n$ are then given by [7]

$$\kappa_n = \frac{2I_n}{m_n^2} = \frac{-8kr_c}{(kx_n\epsilon)^2}\int_0^\pi d\phi\; e^{-2kr_c\phi}\;\phi\chi^{(n)}(\phi), \tag{2.5}$$

where $m_n = kx_n\epsilon$ are the graviton KK excitation masses. Note that we have made use of the gauge freedom discussed in [7] so that $\tilde{S}(t)$, and hence the term proportional to $\sim \partial_\mu\partial_\nu r$, vanishes on the TeV-brane. In this limit the authors of ref. [7] have shown that the "standard" radion interactions are recovered when all SM matter lies on the TeV-brane.

Correspondingly, the radion-dependent parts of the $\mu5$- and 55-components of the metric are given to linear order by the much simpler expressions:

$$\begin{aligned} g^r_{55} &= \frac{\epsilon^{-1}}{\sqrt{6}\pi k r_c M_{pl}} r \,, \\ g^r_{\mu 5} &= \phi \frac{\epsilon^{-1}}{\sqrt{24}\pi k r_c M_{pl}} \partial_\mu r \,. \end{aligned} \tag{2.6}$$

What is interesting about the term arising from the $\mu5$-components of the metric is that it is odd under the $\mathbb{Z}_2$ orbifolding while all zero-mode bulk fields are $\mathbb{Z}_2$ even. As we will see below this implies that any radion couplings induced by these terms must vanish unless accompanied by derivatives of the compactified coordinate. It is clear from the above that the radion couplings to bulk SM fields can arise from terms in the stress-energy tensor of four different types: $T^\mu_\mu$, as above, $T_{\mu\nu}$ contracted with two powers of the radion momenta, $T_{55}$, and lastly $T_{\mu 5}$. With the normalization above the radion couplings to matter that we are interested in are just the product of the $g^r_{AB}$ contracted with the corresponding terms in the stress energy tensor produced by the SM KK zero modes. Let us now examine each of these terms in turn for both massless gauge fields and fermions in the bulk.

### 2.1 Bulk gauge fields

Here we are interested in the decay to and the couplings of on-shell radions to SM gauge bosons. We recall that in the RS model all gauge fields begin as massless in 5-d with spontaneous symmetry breaking taking place on the TeV-brane. The latter contributions to the radion couplings have already been discussed above so here we concentrate solely on the new 5-d pieces ignoring the contributions from SSB. Suppressing possible gauge group indices the general form of the 5-d stress-energy tensor for on-shell (or unitary gauge) *massless* bulk gauge fields is given by [8]

$$T^G_{AB} = \frac{1}{4} g_{AB} F^{MN} F_{MN} - F^C_A F_{BC} \,, \tag{2.7}$$

where Roman letters run over all 5-dimensions. Here, we are only interested in determining the contributions to $T^G_{AB}$ made by the lowest gauge KK mode in the tower since it is this field that we identify with the usual SM gauge boson. To do this end we insert the full KK expansion into the above expression and concentrate on the lowest mode. Besides the usual 4-d pieces, the field strengths $F_{CD}$ in the above expressions involve 4-d derivatives of $A_5$ as well as $\phi$ derivatives of $A_\mu$. Now we recall that ($i$) the fifth component of the gauge field, $A_5$, is *absent* for the zero mode due to the orbifold symmetry and ($ii$) that the zero mode gauge field has a corresponding $\phi$-independent bulk wave function. Employing these conditions it is easy to see that the new *bulk* contributions to both $T^{G\ \mu}_\mu$ and $T^G_{\mu 5}$ must vanish for these zero-modes. Of course it is clear that neither $T^G_{\mu\nu}$ nor $T^G_{55}$ will in general vanish. What about these contributions? Again, using ($i$) and ($ii$) above, all terms proportional to either $A_5$ or $\partial_y A_\mu$ can be dropped; the resulting action induced by the latter term for the KK zero mode is then given by

$$S^G_{55} = \int d^4x dy \sqrt{-g}\, \frac{-r}{\Lambda_r \pi k r_c}\, \frac{-1}{4} \eta^{\mu\nu} \eta^{\lambda\tau} e^{4\sigma} F_{\nu\tau} F_{\mu\lambda}\, \frac{1}{2\pi r_c} \,, \tag{2.8}$$

where the fields strengths are now *only* those of the zero mode. Here we have made use of the notation above and we will further note that in the RS model $g = \det(g_{AB}) = -e^{-8\sigma}$. Upon $y$ integration this term in the action leads to a 4-d interaction of the generic form

$$S_4^{(1)} = \int d^4x \; \frac{r}{\Lambda_r} \frac{1}{4\pi k r_c} F^{\mu\nu} F_{\mu\nu} \tag{2.9}$$

with the indices now contracted by the flat space metric; we will return to the implications of this term later. What about the $T^G_{\mu\nu}$ term? Using the definitions above, the corresponding dimension-7 interaction term arising from $T^G_{\mu\nu}$ and the $\partial_\mu \partial_\nu r$ piece of $g^r_{\mu\nu}$ can be symbolically written (after integration over $y$) as

$$\begin{aligned} S_4^{(2)} = &\int d^4x \; \frac{1}{\pi^2 \sqrt{k r_c} \Lambda_r} \frac{(k/M_{pl})^{-2}}{k r_c \Lambda_\pi^2} \times \\ &\times \sum_n \frac{I_n}{x_n^2} \int_\epsilon^1 \frac{\epsilon^2 dz}{z^3} \left( \frac{z^2 J_2(x_n z)}{J_2(x_n)} - 1 \right) \left[ \partial_\mu \partial_\nu r \eta^{\mu\sigma} \eta^{\nu\lambda} T^G_{\sigma\lambda} \right], \end{aligned} \tag{2.10}$$

where $p^r$ is the radion momentum and we have extracted out only the zero-mode contributions to the stress-energy tensor. Performing the necessary integrations and sums, this reduces to 4-d interaction

$$S_4^{(2)} = \int d^4x \; \frac{1}{\Lambda_r} \left( \frac{k}{0.1 M_{pl}} \right)^{-2} \frac{0.344}{\Lambda_\pi^2} \left[ \partial_\mu \partial_\nu \eta^{\mu\sigma} \eta^{\nu\lambda} T^G_{\sigma\lambda} \right]. \tag{2.11}$$

Note that we expect $k/M_{pl} \lesssim 0.1$ as discussed above so we have scaled the overall numerical factor accordingly. Though the above sum extends over an infinite number of terms we have only included the first two hundred due to its rapid convergence; we have checked that including more terms does not modify our numerical results. We will return to the phenomenological implications of the actions $S_4^{(1)} + S_4^{(2)}$ below.

### 2.2 Bulk fermions

Here we are particularly interested in the decay of radions to SM fermions, i.e. the three-point function for all particles on-shell. Bulk fermions differ from bulk gauge bosons in that a bulk mass term is generally present. SU(2) doublet (D) and singlet (S) SM fields correspond to different states in 5-d with the orbifold symmetry enforcing the correct chirality for the corresponding zero modes. The bulk mass terms for these fields have the form $\mathrm{sign}(\phi) M_D \bar{D} D + (D \to S)$ where the $\mathrm{sign}(\phi)$ factor assures that this term in the action is $\mathbb{Z}_2$-even. (This factor will be suppressed in the discussion that follows.) The fermion tower KK expansions can be written quite generally as $D = \sum D_L^{(n)}(x) \rho^{(n)}(\phi) + D_R^{(n)}(x) \tau^{(n)}(\phi)$ and $S = \sum S_L^{(n)}(x) \tau^{(n)}(\phi) + S_R^{(n)}(x) \rho^{(n)}(\phi)$ where the $\phi$-dependent $\rho(\tau)$ functions are $\mathbb{Z}_2$ even(odd). We now see that the bulk mass terms connect the left- and right-handed states with the same value of $n$ of the form, e.g. $m_n \bar{D}_L^{(n)} D_R^{(n)}$ + h.c. (and similarly for $D \to S$) where $m_n$ is the KK mass. (Note that bulk masses *do not* connect the $S$ and $D$ towers.) Since for the $D(S)$ zero mode only a left-(right-)handed state exists, there is effectively no bulk mass term for this state; hence the bulk mass term will not contribute to the stress-energy tensor for zero modes. By similar arguments it is also clear that all terms containing a $\gamma_5$, the additional gamma matrix in 5-d, must also be absent for zero modes since it too connects terms of opposite helicity but with the same KK number, $n$. All terms containing $\gamma_5$ in the action are of the form $\bar{D}_L^{(n)} D_R^{(n)} + (D \to S) + \text{h.c.}$, but only $D_L^{(0)}$ and $S_R^{(0)}$ exist for zero modes. Thus terms containing both zero modes and $\gamma_5$ are absent. As discussed above, the usual zero mode masses are generated on the TeV-brane through a coupling of the form $\sim \lambda \bar{D} S H \delta(\phi - \pi)$ via SSB; this wall term then provides the "usual" radion coupling to fermions in the RS model. As before, we ignore the effects of SSB when determining the new bulk contributions to the radion couplings. Let us see how these arguments help us to reduce the complexity of the potential bulk-induced fermion couplings to radions.

The general form of the stress-energy tensor for a free massive bulk fermion field can be written as [8]

$$
\begin{aligned}
T^f_{AB} = -g_{AB}\left[\bar{\psi} i \Gamma^C \partial_C \psi - \frac{1}{2}\partial^C \left(\bar{\psi} i \Gamma_C \psi\right) + m\bar{\psi}\psi\right] + \\
+\frac{1}{2}\left[\bar{\psi} i \Gamma_A \partial_B \psi - \frac{1}{2}\partial_B \left(\bar{\psi} i \Gamma_A \psi\right) + (A \leftrightarrow B)\right],
\end{aligned}
\tag{2.12}
$$

where $\Gamma_A$ contains the vielbein and $\psi = D, S$. We now inset the full KK expansion into the equation above and concentrate only on those terms involving the zero mode. The first thing to note is that the terms in the first bracket vanish for zero modes for *all* values of $A, B$. This happens for three reasons: $(i)$ when $C = 5$, a $\gamma_5$ is present which connects $\psi_L^{(0)}$ to $\psi_R^{(0)}$- but only one of these is actually present in the theory due to the orbifold conditions. $(ii)$ A similar argument applies to the contribution from the bulk mass term as it connect left- and right-handed chiral modes. These two contributions are thus zero. $(iii)$ When $C = \mu$ we again get zero by using the bulk equations of motion for the massless zero mode: $i\gamma^\mu \partial_\mu \psi = 0$. Another way to see the vanishing of the first bracket is the use of current conservation for on-shell fields and the full 5-d equations of motion.

The terms in the second bracket remain as potential coupling sources; let us first consider forming $T^{f\ \mu}_{\mu} = 0$ from it. We see immediately that for $A, B = \mu, \nu$ the second bracket vanishes for zero modes when contracted with the 4-d metric due to the bulk equation of motion. Thus we see that the bulk contribution yields $T^{f\ \mu}_{\mu} = 0$ for the zero modes. Next we consider $T^f_{55}$; we see however that when $A, B = 5$ the second bracket vanishes due to the presence of the $\gamma_5$ as discussed above. There remains only the case $T^f_{\mu 5}$ which is somewhat more subtle. In this case $A = \mu, B = 5$, two terms immediately vanish due the presence of a $\gamma_5$ as before. A third term vanishes by using the equations of motion. The last remaining term is then found to be proportional to $\sim \bar{\psi}\gamma_\mu \partial_y \psi$ which doesn't immediately vanish. What form of radion interaction can be obtained from this term? After integration over $y$ and contraction with $g^r_{\mu 5}$, the resulting radion interaction is found to be proportional to $\sim \eta^{\mu\nu}\partial_\nu r \bar{D}_L^{(0)} i\gamma_\mu D_L^{(0)} + D_L \to S_R$. Due to 4-momentum conservation this vanishes for zero mode fermions using the equations of motion as above. In a similar fashion one can analyze the dimension-7 interaction term $\sim \partial_\mu \partial_\nu r \eta^{\mu\sigma}\eta^{\nu\lambda} T^f_{\sigma\lambda}$ which can also be shown to vanish using the Feynman rules in [8] for on-shell radions and massless zero modes. This removes all four potential sources for radion on-shell couplings to zero mode bulk fermions. This implies that the only couplings of radions to fermions arise from the SSB Higgs interaction on the TeV-brane. Thus we conclude that, unlike the case of gauge bosons, on-shell zero mode fermions from the RS bulk KK expansion and fermions on the TeV-brane have identical interactions with radions. The coupling of the radion to SM fermions is insensitive to their location.

## 3. Phenomenology

In the last section we obtained the new couplings of the radion to gauge bosons in the RS bulk and demonstrated that the corresponding terms for fermions are absent. The gauge boson couplings are found to be controlled by the two actions $S_4^{(1)} + S_4^{(2)}$ above, in addition to the brane term for generating masses of the fermions, as well as $W$ and $Z$ bosons, through the Higgs mechanism. The latter brane interaction is the only one present for fermions. To begin, it is instructive to examine the form of momentum space coupling implied by the $\partial_\mu \partial_\nu r \eta^{\mu\sigma}\eta^{\nu\lambda}T^G_{\sigma\lambda}$ term in $S_4^{(2)}$. Labelling the momentum space vertex as $r(p_r)V(k_1, \epsilon_1^\rho)V(k_2, \epsilon_2^\sigma)$ for generic vector bosons $V$ with all momenta flowing into the vertex and using momentum conservation we arrive at the following tensor structure for this piece of the action

$$-p^r_\mu p^r_\nu \eta^{\mu\sigma}\eta^{\nu\lambda}T^G_{\sigma\lambda} = \eta_{\rho\sigma}\left[p_r^2 k_1 \cdot k_2 - 2p^r \cdot k_1 p^r \cdot k_2\right] + k_{1\sigma}k_{2\rho}\left[p_r^2 - 2k_1 \cdot k_2\right], \tag{3.1}$$

where we have used the Feynman rules in [8]. Here we have dropped terms proportional to $k_{1\rho}, k_{2\sigma}$ since we are interested in cases with on-shell gauge bosons or where one of the gauge bosons is virtual and couples to massless fermion pairs. In the case of on-shell radion decay to pairs of gauge bosons the tensor structure for this vertex simplifies to

$$-M_V^2 m_r^2 \eta_{\rho\sigma} + 2M_V^2 k_{1\sigma}k_{2\rho}\,, \tag{3.2}$$

where $M_V$ is the gauge boson mass. (Here we have used the fact that SSB has occurred and set $k_{1,2}^2 = M_V^2$.) Notice that this part of the decay amplitude vanishes when we consider decays to massless gauge bosons. This suggests we examine the massive and massless gauge boson radion decays separately. .

### 3.1 Massive gauge bosons

Combining the terms that arise from $S_4^{(1)} + S_4^{(2)}$ with the usual wall term above [4] we obtain the full $rVV$ coupling; the matrix element for radion decay to massive gauge bosons can be written as

$$\mathcal{M} = \frac{M_V^2}{\Lambda_r}\epsilon_2^\sigma \epsilon_1^\rho \left[A\eta_{\rho\sigma} + \frac{B}{M_V^2}k_{1\sigma}k_{2\rho}\right], \tag{3.3}$$

which leads to the partial decay width

$$\Gamma_V = \frac{M_V^4}{16\pi\ m_r \Lambda_r^2}\left(1 - \frac{4M_V^2}{m_r^2}\right)^{1/2} G(x)\,, \tag{3.4}$$

where $x = k_1 \cdot k_2/M_V^2 = m_r^2/2M_V^2 - 1$ and the function $G$ is given by

$$G(x) = A^2\left(2 + x^2\right) + B^2\left(1 - x^2\right)^2 - 2ABx\left(1 - x^2\right). \tag{3.5}$$

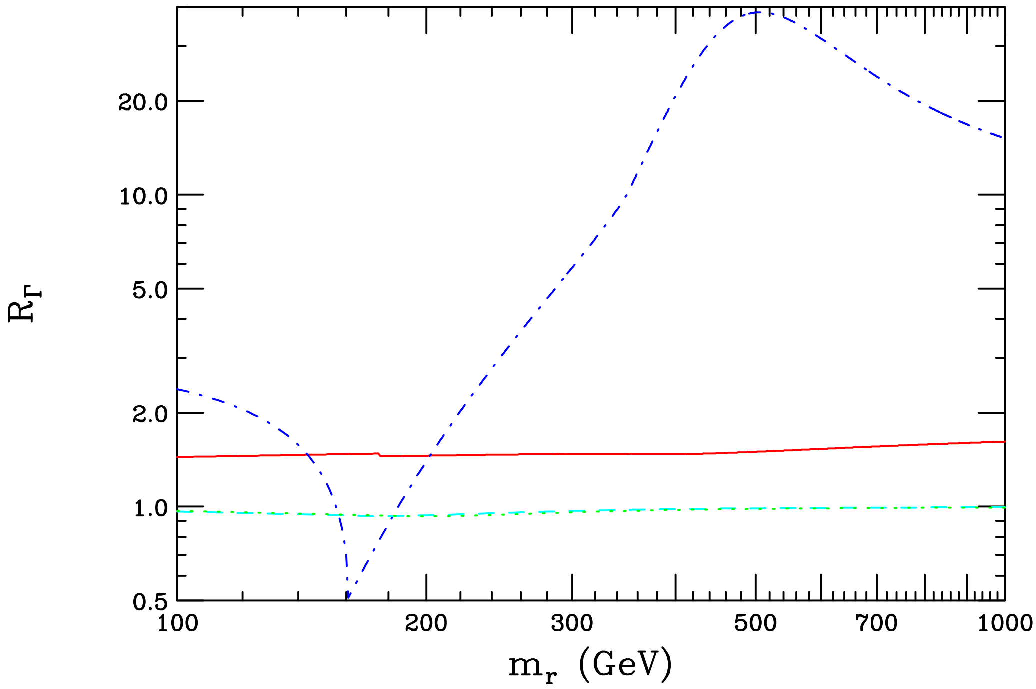


**Figure 1:** The ratios for the partial widths of the radion into SM fields when they are bulk states versus the corresponding case when they are wall states as a function of the radion mass. The solid(dashed-dotted, dashed,dotted) curve is for the $gg(\gamma\gamma, W^+W^-, 2Z)$ final state. $\Lambda_r$, whose value only influences the radion decays to massive gauge bosons, has been taken to be 1.5 TeV along with $k/M_{pl} = 0.1$. No alteration occurs in the case of $\bar{f}f$ or two Higgs boson final states, i.e. $R_\Gamma(\bar{f}f, hh) = 1$.

The dimensionless coefficients $A, B$ are given by

$$A = 1 - \frac{x}{\pi k r_c} - a\frac{m_r^2}{\Lambda_\pi^2}, \qquad B = \frac{1}{\pi k r_c} + 2a\frac{M_V^2}{\Lambda_\pi^2}, \tag{3.6}$$

with $a = 0.344[0.1/(k/M_{pl})]^2$ from above. (In our numerical examples below we will assume that $k/M_{pl} = 0.1$.) We note that in the case of wall gauge fields, $A = 1$ and $B = 0$. From the above expression for $G$ it would at first appear that for arbitrary values of $A, B$ this radion partial width will grow very rapidly as a high power of $m_r/M_V$ as $m_r$ gets large. This is however not the case in reality as these potentially large terms cancel with the specific values for $A$ and $B$ that we obtained above; this provides a test of our results. In fact the growth of this partial width with $m_r$ is found to be no greater than that for the case of wall fields. Taking the ratio of the above partial width to that obtained in the case of gauge fields confined to the TeV-brane, $R_\Gamma$, we obtain the results in figure 1. Here we see that this ratio is almost flat as a function of $m_r$ with a value near $\sim 0.95$ and is found to be quite insensitive to $\Lambda_r$. For the entire mass and parameter range examined, for both the $W^+W^-$ and $ZZ$ final states, this ratio lies well within $\sim$ 5–10% of unity. Clearly these modes alone will not help us test whether or not the SM gauge fields lie in the bulk as these partial widths are essentially unaltered.

### 3.2 Massless gauge bosons

In this case the only new terms arise from $S_4^{(1)}$ and take the same form as those obtained from SM loops and the trace anomaly when the SM fields lie on the TeV-brane [4]. These latter terms will still be present as will those which arise from loops involving the KK excitations of the SM fields. Based on the lower bounds on the KK mass spectrum obtained earlier by Hewett, Petriello and Rizzo [6] and the analysis presented in [9] by Petriello we expect these additional KK loops to give only a very small correction to the usual SM contribution and will be subsequently neglected.

The effective vertex for gluon pairs coupling to the radion is found to be

$$\frac{1}{\Lambda_r}\left(b_3+\frac{2}{\alpha_s kr_c}-\frac{F_g}{2}\right)\frac{\alpha_s}{8\pi}G_{\mu\nu}G^{\mu\nu}r\,, \tag{3.7}$$

where $b_3 = 7$ is the SU(3) $\beta$-function, $G^{\mu\nu}$ is the gluon field strength and $F_g$ is the well-known complex kinematic function of the ratio of masses of the top quark to the radion found in the case of the analogous Higgs boson decay arising from the usual one-loop triangle graph. Similarly, the radion coupling to two photons is now given by

$$\frac{1}{\Lambda_r}\left(b_2+b_Y+\frac{2}{\alpha kr_c}-F_\gamma\right)\frac{\alpha}{8\pi}F_{\mu\nu}F^{\mu\nu}r\,, \tag{3.8}$$

where $b_2 = 19/6$ and $b_Y = -41/6$ are the $\mathrm{SU(2)}\times\mathrm{U(1)}$ $\beta$-functions and $F_\gamma$ is another well-known complex kinematic function of the ratios of the $W$ and top masses to the radion mass arising from the one-loop triangle graphs. In both cases the new terms inversely proportional to $kr_c$ arise from the action $S_4^{(1)}$ and can be numerically quite large since they occur at the tree level. In the case of gluon pairs, the new bulk term increases the $b_3$ contribution by 20–25%. In the case of photon pairs the new contribution is $\simeq 6$ times larger and of the opposite sign than that arising from the beta functions. These new contributions may thus lead to drastic changes in the radion partial widths. Figure 1 shows the ratio of the partial widths for $r \to \gamma\gamma$ and $r \to gg$ for the case of bulk gauge fields to the corresponding ones obtained when these fields are forced to lie on the TeV-brane. As expected we see that the width $r \to gg$ receives a 40–50% increase over the entire radion mass range. The change in the width for $r \to \gamma\gamma$ is much more dramatic, is quite sensitive to the value of $m_r$, and can vary by roughly two orders of magnitude over the relevant mass range.

### 3.3 Collider signatures

The results of figure 1 summarize the differences between radion decays to SM fields when they are in the bulk or on the TeV-brane for various radion partial widths. As far as the $\bar{f}f$, $hh$ and, *essentially*, $W^+W^-$ and $ZZ$ final states are concerned there are *no* differences. Only the $gg$ and $\gamma\gamma$ modes have partial widths which are significantly altered from the TeV-brane case. These changes in the widths directly led to variations in the various branching fractions for decays to bulk fields in comparison to those for wall fields as shown in figure 2. Here we show the deviation of the ratio $R_B = B(r \to \text{bulk})/B(r \to \text{brane})$ from unity for the various final states. Except for the $\gamma\gamma$ case these ratios of branching fractions remain

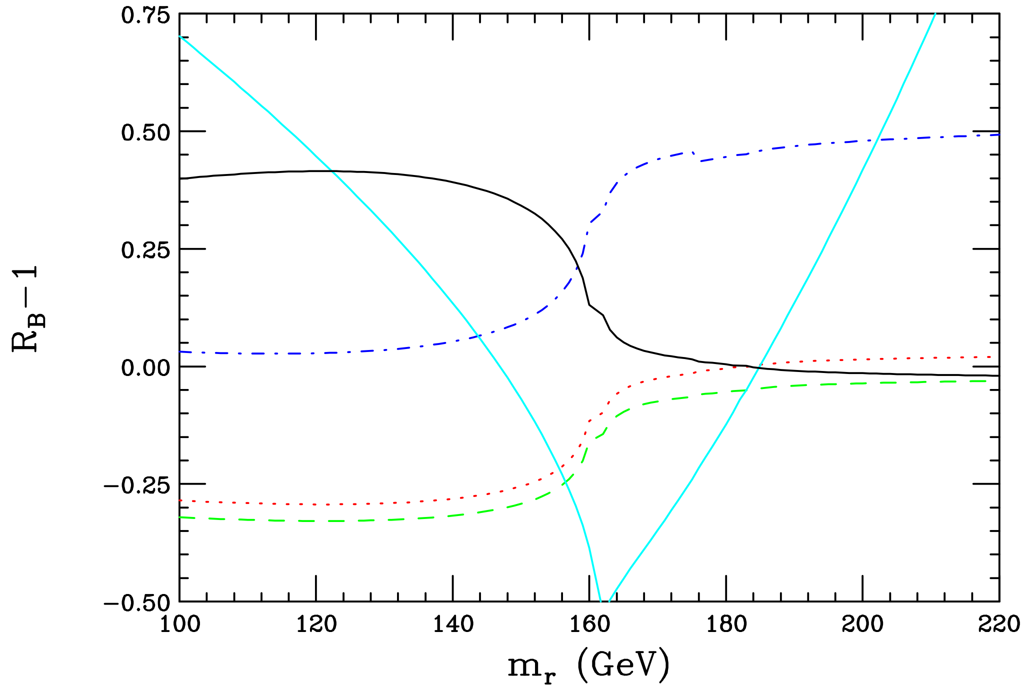


**Figure 2:** The ratios for the branching ratios of the radion into bulk states versus the corresponding wall states as a function of the radion mass. The approximately horizontal curves are, from top to bottom on the left-hand side of the figure, for the total width(solid), and the $gg$(dash-dotted), $\bar{b}b$(dots) and $W^+W^-/ZZ$(dashes) final states. The V-shaped curve is for the $\gamma\gamma$ final state. Radion decay to Higgs boson pairs has been neglected. The values of $\Lambda_r$ and $k/M_{pl}$ are as in the previous figure.

rather flat with increasing $m_r$ above $\simeq 200\,\text{GeV}$. In the case of the $\gamma\gamma$ final state $R_B$ reaches a maximum of $\simeq 40$ for radion masses of order 500 GeV. Knowing both the width and branching fraction changes for all of the decay modes allows one to assess the impact of the "brane vs. bulk" choice for SM matter at colliders. For example, for light radions in the 100 GeV or so range, the production and decay signature at the LHC would be identical to that for a light Higgs, i.e. production by $gg$ fusion followed by decay to two photons. From figures 1 and 2 we see that for radions in this mass range the rate for this process is more that twice as large in the case where the SM fields are in the bulk than when they are on the TeV-brane. On the other hand the production of a radion in association with a SM gauge boson followed by radion decay to $\bar{b}b$ would have a rate at the LHC or Linear Collider which is smaller in the case of bulk SM states by $\sim$ 30–35% in comparison to the case when the SM is on the wall. If light radions are observed at future colliders these small rate differences may help one to determine the locations of the various SM fields in the RS model.

### 3.4 Radion-Higgs mixing

As a last possibility we consider how the mixing of the radion and Higgs field may alter the Higgs boson's couplings in the case when the SM fields are in the bulk. As is well known,



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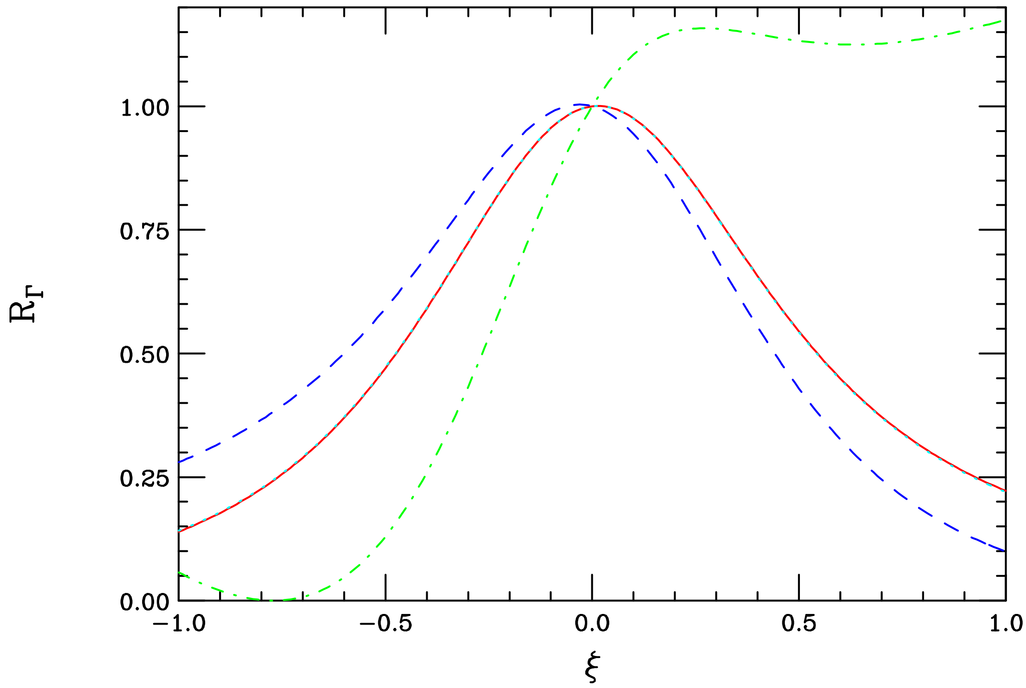


**Figure 3:** The effect of mixing on the partial widths of a 125 GeV Higgs boson, described by the parameter $\xi$, assuming $v/\Lambda_r = 0.2$ and a radion mass of 300 GeV as discussed in the text. The solid(dash-dotted, dashed, dotted) corresponds to the $W^+W^-/2Z(gg, \gamma\gamma, \bar{f}f)$ final states.

on general grounds of covariance, the radion may mix with the SM Higgs field that remains on the TeV-brane through a wall term in the action of the form

$$S_{rH} = -\xi \int d^4x \sqrt{-g_w} R^{(4)}[g_w] H^\dagger H \,, \tag{3.9}$$

where $H$ is the Higgs doublet field, $R^{(4)}[g_w]$ is the Ricci scalar constructed out of the induced metric $g_w$ on the SM-brane, and $\xi$ is a dimensionless mixing parameter assumed to be of order unity and with unknown sign. The above action induces kinetic mixing between the "weak eigenstate" $r_0$ and $h_0$ fields which can be removed through a set of field redefinitions and rotations. This mixing itself is, of course, not directly influenced by whether or not the SM fermion and gauge fields remain on the TeV-brane. Clearly, since the radion and Higgs boson couplings to SM fields differ and the radion couplings depend on the location of the SM fields this mixing will induce modifications in the usual SM expectations for the Higgs decay widths and branching fractions which are sensitive to these various locations.

In earlier work by Hewett and Rizzo [6] the influence of radion-Higgs mixing on the properties of the Higgs were examined in the case where the SM fields were confined to the TeV-brane. Those results will be somewhat modified if instead the SM fields are now placed in the bulk particularly in the case of the $gg$ and $\gamma\gamma$ final states. While a detailed study of this phenomenon is beyond the scope of the present paper we can get an idea of the size of



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this effect by examining the shifts in the Higgs bosons partial widths in these circumstances. These are shown in figure 3 for a typical set of parameter values. Note that these shifts can be significant for values of $\xi$ of order unity or less. Comparison with the results of Hewett and Rizzo, however, show little qualitative difference between the two possible locations of the SM fields. A detailed analysis of these effects will be presented elsewhere.

## 4. Conclusions

In this paper we have examined the couplings of the RS radion to the gauge and fermion fields of the SM when these fields lie in the bulk. These results were then contrasted to the more conventional case where the SM fields are constrained to the TeV-brane. Significant differences in decay widths were found in the case of the $gg$ and $\gamma\gamma$ final states due to the existence of new tree-level terms in the induced 4-d action. Minor differences were also found for the $W^+W^-/ZZ$ final states; the corresponding widths for decays into fermion or Higgs boson pairs were found to be unchanged. These width differences were then shown to lead to relatively large shifts in the overall radion branching fractions.

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## References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[2] For an overview of RS phenomenology, see H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Experimental probes of localized gravity: on and off the wall*, *Phys. Rev.* **D 63** (2001) 075004 [hep-ph/0006041]; *Bulk gauge fields in the Randall-Sundrum model*, *Phys. Lett.* **B 473** (2000) 43 [hep-ph/9911262]; *Phenomenology of the Randall-Sundrum gauge hierarchy model*, *Phys. Rev. Lett.* **84** (2000) 2080 [hep-ph/9909255].

[3] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [hep-ph/9907447]; *Phenomenology of a stabilized modulus*, *Phys. Lett.* **B 475** (2000) 275 [hep-ph/9911457];
C. Csáki, M. Graesser, L. Randall and J. Terning, *Cosmology of brane models with radion stabilization*, *Phys. Rev.* **D 62** (2000) 045015 [hep-ph/9911406];
C. Csáki, M.L. Graesser and G.D. Kribs, *Radion dynamics and electroweak physics*, *Phys. Rev.* **D 63** (2001) 065002 [hep-th/0008151];
C. Charmousis, R. Gregory and V.A. Rubakov, *Wave function of the radion in a brane world*, *Phys. Rev.* **D 62** (2000) 067505 [hep-th/9912160];
T. Tanaka and X. Montes, *Gravity in the brane-world for two-branes model with stabilized modulus*, *Nucl. Phys.* **B 582** (2000) 259 [hep-th/0001092];
P. Kanti, K.A. Olive and M. Pospelov, *On the stabilization of the size of extra dimensions*, *Phys. Lett.* **B 538** (2002) 146 [hep-ph/0204202].



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[4] G.F. Giudice, R. Rattazzi and J.D. Wells, *Graviscalars from higher-dimensional metrics and curvature-Higgs mixing*, *Nucl. Phys.* **B 595** (2001) 250 [hep-ph/0002178];
U. Mahanta and A. Datta, *Production of light stabilized radion at high energy hadron colliders*, *Phys. Lett.* **B 483** (2000) 196 [hep-ph/0002183];
T. Han, G.D. Kribs and B. McElrath, *Radion effects on unitarity in gauge-boson scattering*, *Phys. Rev.* **D 64** (2001) 076003 [hep-ph/0104074];
M. Chaichian, A. Datta, K. Huitu and Z.-h. Yu, *Radion and Higgs mixing at the LHC*, *Phys. Lett.* **B 524** (2002) 161 [hep-ph/0110035];
M. Chaichian, K. Huitu, A. Kobakhidze and Z.H. Yu, *Radions in a $\gamma\gamma$ collider*, *Phys. Lett.* **B 515** (2001) 65 [hep-ph/0106077];
S.B. Bae, P. Ko, H.S. Lee and J. Lee, *Phenomenology of the radion in the Randall-Sundrum scenario at colliders*, *Phys. Lett.* **B 487** (2000) 299 [hep-ph/0002224];
S. Bae, P. Ko, H.S. Lee and J. Lee, *Radion phenomenology in the Randall-Sundrum scenario*, hep-ph/0103187;
S.C. Park, H.S. Song and J. Song, *Z boson pair production at LHC in a stabilized Randall-Sundrum scenario*, *Phys. Rev.* **D 65** (2002) 075008 [hep-ph/0103308];
S.R. Choudhury, A.S. Cornell and G.C. Joshi, *Radion signature in $\gamma\gamma$ scattering*, hep-ph/0012043;
K.-M. Cheung, *Phenomenology of radion in Randall-Sundrum scenario*, *Phys. Rev.* **D 63** (2001) 056007 [hep-ph/0009232];
U. Mahanta, *Implications of radion Higgs couplings for high energy scattering in the Randall-Sundrum model*, *Phys. Rev.* **D 63** (2001) 076006 [hep-ph/0008042];
J.L. Hewett and T.G. Rizzo, *Shifts in the properties of the Higgs boson from radion mixing*, hep-ph/0202155.

[5] G.D. Kribs, *Physics of the radion in the Randall-Sundrum scenario*, hep-ph/0110242.

[6] See, for example, A. Pomarol, *Gauge bosons in a five-dimensional theory with localized gravity*, *Phys. Lett.* **B 486** (2000) 153 [hep-ph/9911294];
Y. Grossman and M. Neubert, *Neutrino masses and mixings in non-factorizable geometry*, *Phys. Lett.* **B 474** (2000) 361 [hep-ph/9912408];
S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, *Bulk standard model in the Randall-Sundrum background*, *Phys. Rev.* **D 62** (2000) 084025 [hep-ph/9912498];
R. Kitano, *Lepton flavor violation in the Randall-Sundrum model with bulk neutrinos*, *Phys. Lett.* **B 481** (2000) 39 [hep-ph/0002279];
S.J. Huber and Q. Shafi, *Fermion masses, mixings and proton decay in a Randall-Sundrum model*, *Phys. Lett.* **B 498** (2001) 256 [hep-ph/0010195]; *Higgs mechanism and bulk gauge boson masses in the Randall-Sundrum model*, *Phys. Rev.* **D 63** (2001) 045010 [hep-ph/0005286];
S.J. Huber, C.-A. Lee and Q. Shafi, *Kaluza-Klein excitations of $W$ and $Z$ at the LHC?*, *Phys. Lett.* **B 531** (2002) 112 [hep-ph/0111465];
J.L. Hewett, F.J. Petriello and T.G. Rizzo, *Precision measurements and fermion geography in the Randall-Sundrum model revisited*, hep-ph/0203091;
F. Del Aguila and J. Santiago, *Signals from extra dimensions decoupled from the compactification scale*, *J. High Energy Phys.* **03** (2002) 010 [hep-ph/0111047]; *Large top mixing from extra dimensions*, hep-ph/0011143; *Top mixing in effective theories*, *Nucl. Phys.* **89** *(Proc. Suppl.)* (2000) 43 [hep-ph/0011142]; *Universality limits on bulk fermions*, *Phys. Lett.* **B 493** (2000) 175 [hep-ph/0008143];

C.S. Kim, J.D. Kim and J.-h. Song, *Top quark Kaluza-Klein mode mixing in the Randall-Sundrum bulk standard model and $B \to X_s \gamma$*, hep-ph/0204002.

[7] A. Das and A. Mitov, *A systematic study of the radion in the compact Randall-Sundrum model*, hep-th/0203205;
See also E.E. Boos, Y.A. Kubyshin, M.N. Smolyakov and I.P. Volobuev, *Effective lagrangians for physical degrees of freedom in the Randall-Sundrum model*, hep-th/0202009.

[8] T. Han, J.D. Lykken and R.-J. Zhang, *On Kaluza-Klein states from large extra dimensions*, *Phys. Rev.* **D 59** (1999) 105006 [hep-ph/9811350];
G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl. Phys.* **B 544** (1999) 3 [hep-ph/9811291].

[9] F.J. Petriello, *Kaluza-klein effects on Higgs physics in universal extra dimensions*, *J. High Energy Phys.* **05** (2002) 003 [hep-ph/0204067].